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Stability Analysis of Marine Ecological Environment Based on Optimal Control of Switched Positive System

Liang Song✉
School of Math and Statistics, Nanyang Institute of Technology
Nanyang 473004
China
Email - songliang255@163.com

Dongyan Lian
School of Math and Statistics, Nanyang Institute of Technology
Nanyang 473004
China

Abstract

In order to analyze the stability of marine ecological environment, a method based on the optimal control of switched positive system is designed and proposed. In this paper, the variational method of control unconstrained and the minimum principle method of control constrained Pontryagin's minimum principle are extended to the optimal switching problem. A two-stage algorithm is proposed to solve the nonlinear optimal problem with fixed switching sequence. In the first stage, the switching time is fixed first, and then the cut-off is obtained under this condition. In the second stage, the optimal solution is obtained by changing the switching time. It should be pointed out that in the first stage, the variational method and the control constrained minimum principle method are extended to the switched system, and then in the second stage, when the switching time is changed, the genetic algorithm is used to obtain the optimal solution, so as to achieve the goal of stability analysis of marine ecological environment. Simulation results show that the proposed method is practical and effective.

Keywords

Switched positive system; Optimal control; Marine ecological environment; Stability analysis.

Introduction

In the actual control problem, due to the physical limitations, it is impossible for the system to operate according to a dynamic system equation all the time, but to switch between several systems. Moreover, several systems often have better performance than a single system. Several unstable systems can get stable switched system through switching (Li et al., 2016). Typical switched systems include automatic transmission system, computer disk drive, some robot control systems, etc. In theory, switched system has also opened up a new field of control theory, which has attracted much attention (Zhao et al., 2016).

The study of nonlinear switched systems is much more complex, involving nonlinear problems, which are generally not easy to be solved thoroughly. At present, relevant scholars have given some good research results, such as the analysis of the optimal control of switched positive system by Ding Hui, but this method will be affected by time and related factors, which will lead to the final analysis results having adverse effects; Li Hui et al. have also analyzed the optimal control of switched positive system, but the operation time of this method is long; Fu Rurong et al. have also proposed a stability analysis and control method of large-scale system using matrix V function. Although the method has achieved good research results, the cost is high. In view of the above problems, a stability analysis method of marine ecological environment based on the optimal control of switched positive system is designed and proposed. The simulation results fully verify the effectiveness and superiority of the proposed method.

Material and Methods

Exponential Stability of Switched Positive System

In the actual control problem, due to the physical limitations, it is impossible for the system to operate according to a dynamic system equation all the time, but to switch between several systems (Zhang and Chen, 2016). Moreover, several systems often have better performance than a single system.

For switching verification system:

$$\tilde{x} = f_i(x), i \in \{1, 2, \cdots, n\} \tag{1}$$

The switching time series s can be expressed in the following forms:

$$t_0 < t_1 < \cdots < t_k < \cdots \lim_{k\to\infty}, t_k = \infty \qquad (2)$$

The switching sequence γ is defined as $\gamma_0 < \gamma_1 < \cdots \gamma_k$, and the following constraints are met (Zheng et al., 2016):

$$\tilde{x} = f_{\gamma k}(x), \Lambda t \in [t_k, t_{k+1}], k = 0,1 \qquad (3)$$

Given a nonlinear autonomous switched system, there are:

$$\tilde{x} = f_i(x), i \in Q = \{1,2,\cdots,k\} \qquad (4)$$

If there is a positive definite radial unbounded function $V(x)$ which makes the function set $\left\{\frac{\partial V}{\partial x}, f_i(x), i \in Q\right\}$ strictly complete on R^n, then the system tends to be stable gradually (Cao, et al., 2016).

A continuous mapping $f(x): R^n \to R$ is assumed to be generalized homogeneous. If there is a positive definite continuous function $\gamma(x,\alpha): R^n \times R^+ \to R$, it is valid for any $\alpha > 0$, $f(\alpha x) = \gamma(x,\alpha) f(x)$.

A generalized homogeneous mapping $f(x)$ has the following properties:

For a certain $x \in R^n$, $f(x) = 0$, the necessary and sufficient condition is for any $\alpha > 0$, $f(\alpha x) = 0$.

A nonlinear programming problem is given as follows:

$$\begin{cases} Max \quad f(x) \\ s.t, s_i \ge 0, i = 1,2,\cdots,m \\ h_j(x) = 0, j = 1,2,\cdots,l \end{cases} \qquad (5)$$

Where $x \in R^n$ is a smooth function, if x^* is the local optimal solution of the nonlinear programming problem, then there are m constants $\lambda_0, \lambda_1, \cdots, \lambda_m$ and l constants $\mu_1, \mu_2, \cdots, \mu_l$ that are not all zero, which make the following set of conditions hold:

$$\begin{cases} \lambda_0 \nabla f(x^*) + \sum_{i=0}^{m} \lambda_i \nabla s_i(x^*) + \sum_{i=0}^{l} \nabla \mu_i \nabla h_i(x^*) = 0 \\ \lambda_i s_i(x^*) = 0, i = 1,2,\cdots,m \\ \lambda_i \ge 0, i = 1,2,\cdots,l \end{cases} \qquad (6)$$

In the above equation, ∇ represents gradient operator (Wu et al., 2016).

To set up a switched system:

$$\tilde{x} = f_i(x), i \in Q = \{1,2,\cdots,n\} \qquad (7)$$

Where $\tilde{x} = f_i(x): R^n \to R^n$ is generally homogeneous, $f_i(0) = 0$, and satisfies all the smooth conditions required in the following derivation (Yu et al., 2016).

If there is a set of positive constant $\{\varepsilon_i, i \in Q\}$ and positive definite symmetric matrix P, which satisfies the completeness of function set $\left\{x^T P f(x)_i + \varepsilon_i \|x\|^2, i \in Q\right\}$, then the system is globally exponentially stable (Wang et al., 2017).

From the positive definite symmetric matrix P, we know that $\lambda_{\min}\|x\|^2 \le x^T P x \le \lambda_{\max}\|x\|^2$, where $\lambda_{\min}$ and $\lambda_{\max}$ are the minimum and maximum eigenvalues of P respectively.

Define switching law as:

$$I(x) = \arg\min\left\{x^T P f_i(x) + \varepsilon_i \|x\|^2\right\} \qquad (8)$$

According to the theory, $I(x) \in R^n \to Q$ is only related to the instantaneous state of the system (Yu et al., 2016). The system under the action of switching law can be described as follows:

$$\tilde{x} = f_{I(x)}(x) \qquad (9)$$

Taking $\varepsilon = \min(\varepsilon_i)$, it is easy to know that the derivative of $V(x) = x^T P(x)$ to t along the orbit of the system needs to meet the following constraints:

$$\tilde{V}(x) \le -\varepsilon \|x\|^2 \qquad (10)$$

It can be seen from the correlation lemma that the system is globally exponentially stable, so the system is globally exponentially stable under the switching law (Tao et al., 2016).

If there are a set of positive constants $(\varepsilon_i, i \in Q)$, a symmetric positive definite matrix P, and $i \in Q = \{1,2,\cdots,n\}$, the following nonlinear programming problems can be solved:

$$\begin{cases} Max\left\{x^T P f_i(x) + \varepsilon_i\right\} \\ s.t \quad x^T P f_i(x) + \varepsilon_i \ge 0, j = 1,2,\cdots,n \\ x^T x = 1 \end{cases} \qquad (11)$$

Stability Analysis of Marine Ecological Environment Based on Optimal Control of

Switched Positive System

The main problem of optimal control research is: according to the state equation of the controlled system, choose an admissible control law, make the controlled object run according to the predetermined requirements, and make a given performance index reach the minimum (or maximum). This belongs to a class of functional extremum problems with constraints (Wang et al., 2016). When the control is unconstrained, that is to say, the admissible control belongs to an open set optimal control problem, it can be solved by classical variational theory; when the control is constrained, it can be solved by Pontryagin's minimum principle or Berman's dynamic programming (Zuo et al., 2016). Generally speaking, any optimal control problem should include the following four aspects:

One, the mathematical model of switched positive system, i.e. the differential equation of the system is as follows:

$$\tilde{x}(t) = f(x,u,t), x(t_0) = x_0 \tag{12}$$

In the above equation, $x(t) \in R^n$ represents the state of switched positive system; $u(t) \in R^n$ represents the control vector; $f(\cdot) \in R^n$ represents the continuous vector function (Li et al., 2016).

Two, boundary condition and target set. The initial state $x(t_0)$ and the final state $x(t_f)$ are the boundary conditions. Generally, the initial time t_0 and the initial state $x(t_0)$ are known. The terminal time t_f and the terminal state $x(t_f)$ can be fixed or free. The target set is the terminal constraint, which is represented as follows:

$$\phi\left[x(t_f), t_f\right] = 0 \tag{13}$$

Three, admissible control. In the actual control problem, most of the control variables can only be taken in a certain range due to the limitation of objective conditions, which is called the control domain. The following uses the Ω flag. The control vector belonging to the control domain is admissible control.

Four, performance index. It is a performance index function to measure the control performance. According to different optimal control problems and different designers, the corresponding performance indexes will be different. Although it is impossible to provide a unified format of performance index for all kinds of optimal control problems, its general form can be summarized as follows:

$$J = \phi\left[x(t_f), t_f\right] + \int_0^L L\left(x(t), u(t), t\right) dt \tag{14}$$

According to the above part, we can generalize the general equationtion of the optimal control problem: under the condition that the controlled system equation and the given initial state are satisfied, an optimal control law $u*(t)$ is determined from the allowable control domain Ω, so that the switched positive system state $x(t)$ starts from the known initial state x_0, and at the terminal time, t_f transfers to the specified target set (13), and make the performance index (14) reach the extreme value. Generally, the optimal control problem can be expressed in the following functional form:

$$\begin{cases} \min\limits_{u(t)\in J} = \varphi\left[x(t_f), t_f\right] + \int_0^{t_f} L\left[x(t), u(t), t\right] dt \\ s.t. \tilde{x}(t) = f\left[x(t), u(t), t\right], x(t_0) = x_0 \\ \phi\left[x(t_f), t_f\right] = 0 \end{cases} \tag{15}$$

When the control vector is unconstrained and time continuous, the necessary conditions of the optimal control solution can be derived by the variational method. In the variational method, the case of composite index functional, end constrained and fixed t_f at the end time is the most representative. The necessary conditions for this kind of optimal control problem are given below (Niu et al., 2018). This kind of problem is a functional extremum problem with equality constraints. Lagrange multiplier method can be used to simplify the constrained functional extremum problem to unconstrained functional extremum problem (Gao et al., 2018). The Hamiltonian function is constructed as follows:

$$H(x,u,\lambda,t) = L(x,u,t) + \lambda^T(t) f(x,u,t) \tag{16}$$

For the following optimal control problems:

$$\min_{u(t)} J = \varphi\left[x(t)\right] + \int_0^t L(x,u,t) dt \tag{17}$$

$$s.t \tilde{x}(t) = f(x,u,t), x(t_0) = x_0, \phi\left[x(t_f)\right] = 0 \tag{18}$$

Genetic algorithm (GA) is a random search algorithm simulating Darwin's natural selection and genetic mechanism. It simulates the problem to be solved into a process of biological evolution, generates the next generation through selection, crossover, mutation and other operations, and gradually eliminates the individuals with low adaptability, making the whole body more and more adaptable. The whole genetic algorithm includes parameter coding, population initialization, fitness function selection, selection, crossover, mutation and stop criteria setting.

Next, the process of solving the optimal control problem of the switched positive system in the second stage with genetic algorithm is analyzed. When genetic algorithm is used to deal with the optimal control problem of the switched system in the second stage, the time interval is $\left[t_0, t_f\right]$, where the switching time is an independent variable. First, binary coding is carried out for it.

$$s \in \left[t_0, t_f\right], b \in Binary\left(L\right), s = t_0 + \frac{b}{2^L - 1} * \left[t_0 - t_f\right] \quad (19)$$

The second step is to initialize the population and randomly generate the binary numbers of L bits in $M \quad Binary\left(L\right)$. Then the M data is used as the initial point to start the iteration. The larger M is, the larger the search range is. However, the longer the genetic operation time of each generation is, the smaller M_2 is, the smaller the search range is, and the shorter the genetic operation time of each generation is. Generally, $M = 20 \times 100$ is selected.

In the third step, the fitness function is selected. When the genetic algorithm is used to deal with the optimal control problem of the switched system in the second stage, the fitness function is taken as $J\left(s\right) = \varphi\left[x\left(t_f\right)\right] + \sum_{k=0}^{K} \int_k^{k+1} \left(L\left(x,u\right) + p^T\left(t\right)\right)$, which is obtained by solving the boundary differential equations at both ends formed by the necessary conditions of the optimal control problem of the switched system in the first stage.

In this part, the optimal problem of switched nonlinear system with unlimited control is introduced firstly, and then the stability analysis problem of marine ecological environment based on the optimal control switched positive system is studied more deeply, and a kind of solution is given. Because the general optimal control problem of switched systems is still difficult to solve, it is necessary to fix the switching order and times in advance, and then introduce a two-stage algorithm. In the first stage, we first fix the switching time, extend the variational method for control unconstrained and the minimum principle method for control constrained to the switched system, and then use the extended method to obtain the optimal solution of the optimal control problem of the switched system; in the second stage, the switching time is changed, and genetic algorithm is used to obtain the optimal solution. Using genetic algorithm does not need to consider the objective function, so that the stability analysis of marine ecological environment based on the optimal control of switched positive system is realized.

Results

In order to verify the comprehensive effectiveness of the proposed method for the stability analysis of marine ecological environment based on the optimal control of switched positive system, simulation experiments are needed. The experimental environment is as: Pentium 4 CPU with dual core, 3.2 GHz, 3 GB memory, and the software to realize the algorithm is Matlab7.0.1.

One, operating efficiency / (%):

In order to verify the effectiveness of various analysis methods, the following simulation experiments are carried out with operation efficiency as the evaluation index, and the comparison results of operation efficiency of the three methods are shown in the following figure:

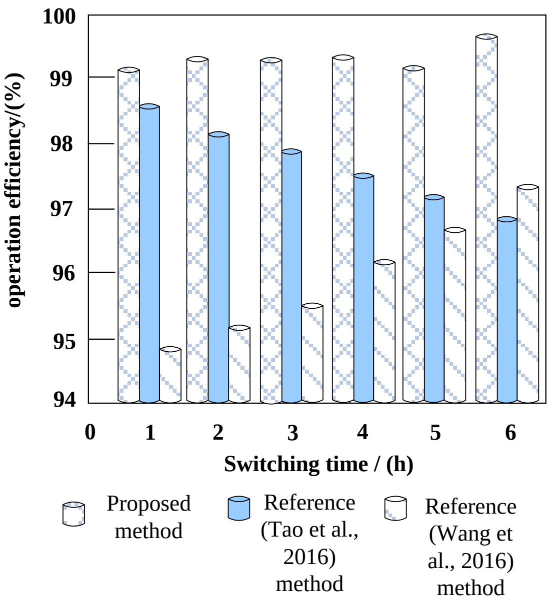

Figure 1: Comparison results of operation efficiency of different methods.

Comprehensive analysis of the experimental data in Figure 1 shows that the operation efficiency of the proposed method is the highest among the three methods, which shows that the proposed method can effectively reduce the operation time of the whole method and improve the operation efficiency by using genetic algorithm for optimization.

Two, cost (Yuan):

The cost of different analysis methods is also a hot topic that has always been studied. The cost comparison results of the three methods are shown in Table 1.

According to Table 1, the cost of the proposed method is the lowest among the three methods; the cost of the method in reference (Tao et al., 2016) is the second; the cost of the method in reference (Wang et al., 2016) is the highest. The reason is that after selection, crossover and mutation, the proposed method selects the individuals with high fitness as the next generation. After several generations, until the termination conditions set by genetic algorithm are met, the solution of the whole switching nonlinear optimal problem is obtained, and the cost is reduced to a certain extent.

Table 1: Cost changes of three methods

Number of experiments	Cost of the proposed method / (Yuan)	Cost of the method in reference (Tao et al., 2016) / (Yuan)	Cost of the method in reference (Wang et al., 2016) / (Yuan)
5	1587	1885	1978
10	1654	1974	2041
15	1787	2014	2121
20	1842	2178	2225
25	1917	2258	2341
30	2085	2369	2492
35	2126	2447	2574
40	2287	2521	2615
45	2396	2685	2774
50	2441	2774	2884
55	2578	2887	2979
60	2625	2985	3058
65	2795	3045	3199
70	2847	3135	3260
75	2985	3274	3387
80	3032	3396	3491
85	3125	3485	3587
90	3241	3596	3647
95	3356	3678	3796

Conclusion

Based on the existing theoretical results of switched system, this paper studies the stability analysis of marine ecological environment based on the optimal control of switched positive system

One, in the first stage, the optimal control problem of switched nonlinear system is solved by fixing the switching time; Two, in the second stage, the optimal solution is obtained by changing the switching time. It should be pointed out that in the first stage, the variational method for control unconstrained and the minimum principle method for control constrained are extended to the switched system, and then in the second stage, the genetic algorithm is used to obtain the optimal solution when the switching time is changed, without considering the derivative solution of the objective function with the switching time as the independent variable about the switching time, so as to achieve the stability analysis of the marine ecological environment.

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